bon units; μ_{He}, atomic mass of an He molecule, carbon units; μ_W, atomic mass of W atom, carbon units; c, condensation coefficient, dimensionless; P, gas pressure, mm Hg; m, cooling rate of tungsten filament, 1/sec; X, fraction of tungsten covered with contaminants, dimensionless.

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"EQUILIBRIUM" APPROXIMATIONS IN THEORY OF RADIATIVE

HEAT TRANSFER

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Two methods are proposed for approximately taking into account the selectivity and anisotropy of radiation characteristics in calculation of radiative heat transfer. Both methods are shown to be sufficiently reliable and effective.

Accounting for selectivity and anisotropy is the most difficult problem in calculations and studies of radiative heat transfer. It therefore is important to develop approximate but still sufficiently accurate and reliable methods of calculating radiative heat transfer with these features taken into account. Such methods can be developed along two main lines. In the first line, the approach is based on simplifying assumptions regarding the radiation characteristics of bodies and media which participate in the heat transfer (selectively gray, gray, diffuse, diffuse-specular, or other approximations). In the second line of development, one makes no assumptions pertaining to the radiation characteristics of materials, but accepts some premises regarding the characteristics of the radiation which interacts with the bodies and the media within a given system.

In this study, two approximations will be considered along the second line, called "equilibrium" approximations on account of the assumed equilibrium distribution of spectral intensity with respect to any integral radiation characteristics.

Approximation of "Equilibrium" Incident Radiation Density. In calculation of heat transfer by this method, it is possible to approximately account for the selectivity and also (partly) the anisotropy of radiation characteristics. One assumes here that the distribution of spectral intensity with respect to volume density and surface density of the incident radiation is in equilibrium and obeys Planck's law at a certain radiation temperature. This radiation temperature is, in turn, determined by the density of the incident integral radiation. On the basis of this approach, the integral absorptivity and reflectivity of bodies, as well as the irradiance coefficients (appearing in the theoretical equations), will depend not only on the temperature of a body but also on the density of the incident integral radiation. These relations can be established beforehand, however, if the geometry of the system and the radiation characteristics of the bodies participating in the heat transfer are known. The integral emissivity of bodies in a fixed direction, moreover, depends only on their physical temperature and can be determined without particular difficulties. By using this approximation in algebraic methods of heat transfer calculation, one can obtain a system of nonlinear algebraic equations which, approximately, take into account both the selectivity and the anisotropy of radiation. This resulting system of equations can be solved by the method of iterations without any fundamental difficulties.

Let us examine in detail the approximation of equilibrium density of incident radiation, using as an example the zone method of heat transfer calculation. The radiating system will be assumed to consist of N surface zones separated by a diathermal medium. According to the conventional formulation of the problem, for some zones ($k = 1, 2, \ldots, N_1$) the distribution of surface temperature (T_w) will be given, and for the remaining zones ($k = N_1 + 1, N_1 + 2, \ldots, N$) the surface density of resultant radiation (E_r) will be given. The system of algebraic equations based on approximation of the fundamental integral equation and yielding the density of incident radiation E_{in}, in the given formulation, is

$$\sum_{k=1}^{N} A_{ik} E_{\mathrm{in},k} = \sum_{k=1}^{N_1} \varphi_{ik} \varepsilon_k E_{\mathrm{r},k}^{(\mathrm{giv})} - \sum_{k=N_1+1}^{N} \varphi_{ik} E_{\mathrm{r},k}^{(\mathrm{giv})}, \quad i = 1, 2, \ldots, N, \tag{1}$$

with the coefficients A_{jk} determined from the relations

$$A_{ik} = \begin{cases} 1 - \varphi_{ii} r_i, & i = k, \ 1 \leqslant i \leqslant N_1, \\ 1 - \varphi_{ii}, & i = k, \ (N_1 + 1) \leqslant i \leqslant N, \\ (-\varphi_{ik} r_k), & i \neq k, \ 1 \leqslant k \leqslant N_1, \\ (-\varphi_{ik}), & i \neq k, \ (N_1 + 1) \leqslant k \leqslant N. \end{cases} \tag{2}$$

This system of algebraic equations (1) is a nonlinear one, inasmuch as the irradiance coefficients φ_{ik} and the reflection coefficients r_k (which determine the $\|A\|$ matrix) depend on both the temperature $T_{w,k}$ of a given zone and the surface density of incident radiation within that zone $E_{\mathrm{in},k}$. An analysis reveals that these quantities can be determined from the relations

$$\varphi_{ik} = \varphi_{ik}(T_{w,k}, E_{\mathrm{in}k}) = \frac{F_k}{F_i} \times$$

$$\times \left[\int_{\nu=0}^{\infty} E_{0,\nu}(\nu, T_{w,k}) \, d\nu \int_{(F_k)} dF_P \int_{(F_i)} a_{\nu,s}(P, s_{MP}) K(P, M) \, dF_M + \right.$$

$$+ \int_{\nu=0}^{\infty} E_{0,\nu}(\nu, T_{\mathrm{r},k}) \, d\nu \int_{(F_k)} dF_P \times$$

$$\times \left. \int_{(F_i)} r_{\nu,s}(P, s_{MP}) K(P, M) \, dF_M \right] \Big/ \left[\int_{\nu=0}^{\infty} E_{0,\nu}(\nu, T_{w,k}) \, d\nu \int_{(F_k)} a_\nu(P) \, dF_P + \right.$$

$$+ \left. \int_{\nu=0}^{\infty} E_{0,\nu}(\nu, T_{\mathrm{r},k}) \, d\nu \int_{(F_k)} r_\nu(P) \, dF_P \right], \tag{3}$$

$$P \in F_k; \ M \in F_i; \ i, k = 1, 2, \ldots, N,$$

$$r_k = r_k(T_{w,k}, E_{\mathrm{in}k}) = \frac{\displaystyle\int_{\nu=0}^{\infty} E_{0,\nu}(\nu, T_{\mathrm{r},k}) \, d\nu \int_{(F_k)} r_\nu(P) \, dF_P}{E_{\mathrm{in}k} F_k}, \tag{4}$$

$$P \in F_k; \ k = 1, 2, \ldots, N.$$

The zonal absorption coefficients a_k depend on the same quantities $T_{w,k}$ and $E_{in,k}$, being related to the reflection coefficients r_k through the well-known equality

$$a_h = a_h(T_{w,h}, E_{in,h}) = 1 - r_h(T_{w,h}, E_{in,h}), \quad h = 1, 2, \ldots, N, \tag{5}$$

where $r_k(T_{w,k}, E_{in,k})$ are determined from relation (4).

The integral hemispherical emission coefficients ε_k for each zone are functions of the zone temperature $T_{w,k}$ only, and can be calculated according to the expression

$$\varepsilon_k = \varepsilon_h(T_{w,h}) = \frac{\int\limits_{v=0}^{\infty} E_{0,v}(v, T_{w,h}) \, dv \int\limits_{(F_h)} a_v(P) \, dF_P}{\sigma_0 T_{w,h}^4 F_h}, \tag{6}$$

$$P \in F_h; \quad k = 1, 2, \ldots, N.$$

In this way, the specific form of relations $\varphi_{jk}(T_{w,k}, E_{in,k})$, $r_k(T_{w,k}, E_{in,k})$, and $\varepsilon_k(T_{w,k})$ for a given radiating system can be established beforehand on the basis of expressions (3), (4), and (6).

Since for zones $k = N_1 + 1$, $N_1 + 2$, $\ldots$, N the surface densities of resultant radiation $E_{r,k}$ are given while their temperatures $T_{w,k}$ are unknown, according to the conventional statement of the problem, it is necessary to add to the fundamental system of Eqs. (1) equations which relate $E_{r,k}$ and $E_{in,k}$

$$E_{\text{T},h} = \frac{1}{\varepsilon_h}(a_h E_{in\,h} - E_{r,h}^{(\text{giv})}), \quad k = N_1 + 1, \ldots, N. \tag{7}$$

Upon combining the two systems (1) and (7), one arrives at the expressions

$$E_{\text{in}\,i}^{(n+1)} = \sum_{k=1}^{N_1} \frac{\varphi_{ih}(T_{w,h}^{(\text{giv})}, E_{\text{in}\,h}^{(n)})}{A_{ii}^{(n)}} \varepsilon_h(T_{w,h}^{(\text{giv})}) E_{\text{T},h}^{(\text{giv})} -$$

$$- \sum_{h=N_1+1}^{N} \frac{\varphi_{ih}(T_{w,h}^{(n)}, E_{\text{in}\,h}^{(n)})}{A_{ii}^{(n)}} E_{\text{T},h}^{(\text{giv})} - \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{A_{ih}^{(n)}}{A_{ii}^{(n)}} E_{\text{in}\,h}^{(n)}, \quad i = 1, 2, \ldots, N, \tag{8}$$

$$T_{w,i}^{(n+1)} = \left[\frac{1}{\sigma_0 n^2 \varepsilon_i(T_{w,i}^{(n)})}(a_i(T_{w,i}^{(n)}, E_{\text{in}\,i}^{(n)}) E_{\text{in}\,i}^{(n)} - E_{r,i}^{(\text{giv})}) \right]^{\frac{1}{4}}, \quad i = N_1 + 1, \ldots, N, \tag{9}$$

for determining the sought quantities in the $(n + 1)$-th approximation.

Solution of the system (1) and (7) by the method of iterations yields the surface densities of incident radiation $E_{in,k}$ ($k = 1, 2, \ldots, N$) in all zones and the sought temperatures $T_{w,k}$ ($k = N_1 + 1, \ldots, N$) in the zones where $E_{r,k}$ was given. It is now easy to determine the surface densities of resultant radiation $E_{r,k}$ ($k = 1, 2, \ldots, N_1$) in the zones; temperatures were stipulated, namely by using the well-known expression

$$E_{\text{r},k} = a_h E_{\text{n},h} - \varepsilon_h(T_{w,h}^{(\text{giv})}) E_{\text{T},h}^{(\text{giv})}, \quad k = 1, 2, \ldots, N_1. \tag{10}$$

Approximation of Equilibrium Intensity. The approximation of equilibrium intensity is distinguished by a high accuracy, in comparison with the preceding approximation. This is attributable to the fact that this approximation refers to finer characteristics, namely to the integral radiation intensities with the spectral distribution assumed to be in equilibrium and assumed to obey Planck's law at the corresponding radiation temperature. The latter is, in turn, determined by the integral intensity within the given spectral range. The emissivity of a surface and the emissivity of a medium depend in this approximation on their temperatures, while the integral radiation characteristics which determine the absorption as well as volume and surface scattering are functions of both the local temperature and the integral intensity. When the spectral radiation characteristics of bodies and media in the radiating system are known, then the relations for all integral radiation characteristics can be determined as before. Since the equations of transfer and boundary conditions in the approximation of equilibrium intensity are written in terms of integral intensity, this greatly simplifies the solution of the problem, on account of obviating the need for analysis of the spectral equations and subsequent integration over the spectrum.

The entire radiation spectrum is subdivided into bands characteristic of individual segments (zones) of the system, within each of which the radiation characteristics vary relatively little. For instance, under certain conditions such a segment can cover the entire frequency spectrum. Integrating the spectral equation of radiation transfer with respect to frequency over each spectral band, we obtain for each segment the system of equations of radiation transfer

$$s \cdot \nabla I_i(s) = j_i^{(V)} - k_i I_i(s) +$$
$$+ \frac{1}{4\pi} \int\limits_{(4\pi)} \rho_i^{(V)}(s', s) I_i(s') d\omega_{s'}, \quad i = 1, 2, \ldots, N. \tag{11}$$

Analogous integration of the spectral equations describing the boundary conditions yields for the boundary surface the system of equations of integral radiation

$$I_{i,-n}(s) = j_i^{(F)}(s) + \frac{1}{\pi} \int\limits_{(2\pi_{+n})} I_{i,+n}(s') \cos(s', n) \rho_i^{(F)}(s', s) d\omega_{s'}, \quad i = 1, 2, \ldots, N. \tag{12}$$

In Eqs. (11) and (12) the quantity $I_i(s)$ represents the integral radiation intensity within the i-th band of the spectrum, determined through integration of the spectral intensity $I_\nu(s)$

$$I_i(s) = \int\limits_{\nu_{min, i}}^{\nu_{max, i}} I_\nu(s) \, d\nu. \tag{13}$$

The total radiation intensity over the entire frequency spectrum I(s) is calculated by summation of the integral intensities over all bands

$$I(s) = \sum_{i=1}^{N} I_i(s). \tag{14}$$

The volume coefficients $\rho_i^{(V)}$ and the surface coefficients $\rho_i^{(F)}$ of brightness, as well as the attenuation coefficients k_i for each band, depend on the local temperature of the medium T or of the body surface T_w, as well as on the integral intensity I_i, and can be evaluated on the basis of the quadrature relations

$$\rho_i^{(V)}(s', s) = \rho_i^{(V)}(T, I_i; s', s) =$$
$$= \frac{1}{I_i} \int\limits_{\nu_{min, i}}^{\nu_{max, i}} \rho_\nu^{(V)}(T; s', s) n_\nu^2 I_{0,\nu}(\nu, T_{p,i}(I_i)) d\nu, \tag{15}$$

$$\rho_i^{(F)}(s', s) = \rho_i^{(F)}(T_w, I_i; s', s) =$$
$$= \frac{1}{I_i} \int\limits_{\nu_{min, i}}^{\nu_{max, i}} \rho_\nu^{(F)}(T_w; s', s) n_\nu^2 I_{0,\nu}(\nu, T_{p,i}(I_i)) d\nu, \tag{16}$$

$$k_i = k_i(T, I_i) = \frac{1}{I_i} \int\limits_{\nu_{min, i}}^{\nu_{max, i}} k_\nu'(T, \nu) n_\nu^2 I_{0,\nu}(\nu, T_{p,i}(I_i)) d\nu, \quad i = 1, 2, \ldots, N, \tag{17}$$

with the radiation temperature $T_{r,i}$ (i = 1, 2, ..., N) for each spectral band determined by the corresponding integral intensity I_i for that range in accordance with the equation

$$I_i = \int\limits_{\nu_{min, i}}^{\nu_{max, i}} n_\nu^2 I_{0,\nu}(\nu, T_{p,i}) d\nu. \tag{18}$$

The volume coefficients $j_i^{(V)}$ and the surface coefficients $j_i^{(F)}$ of intrinsic radiation are determined solely by the intrinsic local temperatures of the medium T and of the surface T_w

$$j_i^{(V)} = j_i^{(V)}(T) = \int\limits_{\nu_{min, i}}^{\nu_{max, i}} \alpha_\nu'(\nu, T) n_\nu^2 I_{0,\nu}(\nu, T) d\nu, \tag{19}$$
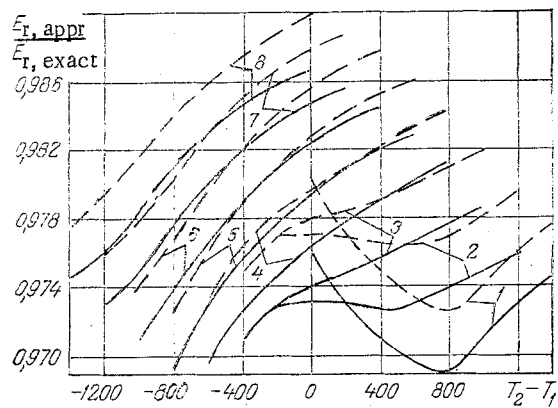
Fig. 1. Comparison of results based on equilibrium approximations with numerical solution of the problem in exact formulation: 1) $T_1 = 1200°K$; 2) 1400; 3) 1600; 4) 1800; 5) 2000; 6) 2200; 7) 2400; 8) 2600°K.

$$j_i^{(F)} = j_i^{(F)}(T_w, \; \mathbf{s}) = \int\limits_{v_{min,\, i}}^{v_{max,\, i}} a_{v,s}(\mathbf{s}, \; T_w) n_v^2 I_{0,v}(v, \; T_w) dv, \quad i = 1, \, 2, \ldots, \, N. \tag{20}$$

The radiation characteristics of the medium and of the body surface $\rho_i^{(V)}(\mathbf{s}', \, \mathbf{s})$, $\rho_i^{(F)}(\mathbf{s}', \, \mathbf{s})$, $k_i(T, \, I_i)$, $j_i^{(V)}(T)$ and $j_i^{(F)}(T_w, \, \mathbf{s})$ are calculated beforehand for each spectral band on the basis of relations (15)-(17), (19), and (20), and their values are inserted into the system of Eqs. (11)-(12), which is then solved by numerical methods with Eq. (18) taken into account.

Application of Given Approximate Methods. For the purpose of estimating the accuracy of both approximations, they were used for solving the problem of radiative heat transfer in a system of parallel optically smooth surfaces with each at a different temperature. The results were compared with the numerical solution to the same problem in the exact formulation, taking into account both selectivity and anisotropy [1]. The results of such a comparison are shown in Fig. 1. For tungsten as the material of both surfaces, their temperatures (T_1 and T_2) are, respectively, T_1, $T_2 = 1200, 1400, 1600, 1800, 2000, 2200, 2400,$ and $2600°K$. The temperature difference $T_2 - T_1$ has been selected as argument of the relation shown here, and the temperature T_1 of the first surface has been selected as its parameter (curves 1-8). Along the axis of ordinates we have plotted the ratio of the surface density of resultant radiation, obtained on the basis of some equilibrium approximation ($E_{r,appr}$), to a similar value based on the numerical solution to the problem in the exact formulation ($E_{r,exact}$). The solid lines refer to the approximation of equilibrium density of incident radiation, the dashed lines refer to the approximation of equilibrium radiation intensity.

The graphs indicate, first of all, that both approximations are sufficiently accurate (the maximum error only slightly exceeds 3%). As was to be expected, the approximation of equilibrium radiation intensity is more accurate than that of equilibrium density of incident radiation. According to the graphs, however, the difference in accuracy between the results of the two approximations is small, which can be explained by the geometrical characteristics of the given radiating system.

NOTATION

$E_{T,k} = n^2 \sigma T_{w,k}^4$ is the surface density of equilibrium radiation at the surface temperature $T_{w,k}$ of the k-th zone in a medium with a refractive index n; F_i and F_k, surfaces of the i-th zone and the k-th zone, respectively; $T_{r,k} = (E_{in,k}/\sigma_0 n^2)^{1/4}$, radiation temperature corresponding to the surface density of incident radiation for the k-th radiation zone; $T_{w,k} = \left(\dfrac{1}{F_k} \int\limits_{(F_k)} T_w^4(P) dF_P\right)^{1/4}$, mean surface temperature of the k-th zone $(P \in F_k)$; $E_{0,v}(v, \, T)$, spectral surface density of equilibrium radiation in vacuum at frequency v and temperature T; $a_{v,s}$-

(P, s_{MP}) and $r_{\nu,s}$(P, s_{MP}) = $1 - a_{\nu,s}$(P, s_{MP}), spectral directional absorption coefficients and reflection coefficients of the surface at point P for radiation at frequency ν coming from point M; K(P, M) = $(\cos \theta_P \cos \theta_M)/\pi r^2_{MP}$, kernel of the integral equation of heat transfer, known from theory and a function of points M, P in the given system; $a_\nu(P) = \dfrac{1}{\pi} \displaystyle\int_{(2\pi)} a_{\nu,s}(P, s)$

$\cos(s, n_p)d\omega_s$, $r_\nu(P) = 1 - a_\nu(P)$, respectively, spectral hemispherical absorption coefficient and reflection coefficient of the surface at point P for ideal diffuse radiation at frequency ν; $E^{(giv)}_{r,k}$, $T^{(giv)}_{w,k}$ and $E^{(giv)}_{T,k}$, quantities as stipulated; $E^{(n)}_{in,i}$ and $E^{(n+1)}_{in,i}$, $T^{(n)}_{w,i}$ and $T^{(n+1)}_{w,i}$, quantities defined in n-th and (n + 1)-th approximations of the iteration process; $d\omega_{s'}$, element of the solid angle about direction s'; s', unit vector in the given direction; $I_{i-n}(s)$, integral intensity of effective radiation within the i-th band in direction s; .n, unit vector normal to the boundary surface at a given point; $\nu_{min,i}$ and $\nu_{max,i}$, respectively, lower limit and the upper limit of the i-th band; $\rho_\nu^{(V)}(T; s', s)$ and $\rho_\nu^{(F)}(T_w, s', s)$, spectral coefficients of brightness of the medium and for the surface, respectively; $k_\nu'(T, \nu)$ and $\alpha_\nu'(T, \nu)$, respectively, spectral attenuation coefficient and spectral absorption coefficient for the given medium; n_ν, spectral refractive index for that medium; and $I_{o,\nu}(\nu, T)$, spectral intensity of equilibrium radiation in vacuum.

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TEMPERATURE DISTRIBUTION IN THIN FILMS RECEIVING
RADIANT ENERGY

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An expression is derived for estimating the temperature distribution in thin metal and semiconductor films locally receiving radiant energy.

The operating modes and the design configurations of thin-film devices are known to depend on the number and the characteristics of influencing external factors (electric current, magnetic field, temperature, etc.). Thermal action is of special significance among such factors, inasmuch as, even when none is assumed to take place, temperature fields can build up as a result of conversion of other forms of energy to heat.

It has been estimated in earlier studies [1-3] how the thermal (superposing) galvanothermomagnetic effects in semiconductor bulk specimens influence the accuracy of determination of the underlying galvanothermomagnetic effects, and the feasibility of determining several thermophysical parameters of specimens on the basis of complete separation of all measurable effects has been established. As far as these authors know, no such study was ever made with regard to film specimens.

Other studies [4-6] have dealt with the temperature distribution and the thermal fluxes in thin films resulting from action of laser radiation. It has been established that, as a rule, the thermal flux Φ_2 to the substrate exceeds the thermal flux Φ_1 in the plane of the film (except in two cases: during a short initial time period of the order of 10^{-8} sec, and when the substrate has a low thermal conductivity).

In some problems, such as recording the distribution of energy flux density in the focal spot of solar concentrators [7, 8], or calculating the thermal distortion of the contour of the laser irradiation zone [4-6, 9], or determining the buildup time of superposing galvano-